On the global structure of self-gravitating disks for softened gravity

Jesper Sommer-Larsen[1], Henrik Vedel[1] and

Uffe Hellsten[2]

[1]Theoretical Astrophysics Center
Juliane Maries Vej 30, DK-2100 Copenhagen Ø, Denmark
(jslarsen@tac.dk, vedel@tac.dk)

[2]University of California, Lick Observatory,
Santa Cruz, CA 95064, USA
(uffe@ucolick.org)

Abstract

Effects of gravitational softening on the global structure of self-gravitating disks in centrifugal equillibrium are examined in relation to hydrodynamical/gravitational simulations. The one-parameter spline softening proposed by Hernquist & Katz is used.

It is found that if the characteristic size of a disk, r, is comparable to or less than the gravitational softening length, ϵ, then the cross section of the

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simulated disk is significantly larger than that of a no-softening (Newtonian) disk with the same mass and angular momentum.

We furthermore demonstrate that if $r \lesssim \epsilon/2$ then the scaling relation $r \propto \epsilon^{3/4}$ holds for a given mass and specific angular momentum distribution with mass. Finally we compare some of the theoretical results obtained in this and a previous paper with the results of numerical Tree-SPH simulations and find qualitative agreement.

Key words: galaxies: kinematics and dynamics – galaxies: structure – methods: numerical

1 Introduction

In pure N-body as well as gravitational/hydrodynamical, particle-based simulations the gravitational field of the individual particles is commonly softened, primarily to suppress effects of two-body gravitational interactions - see, e.g., Evrard (1988), Hernquist & Katz (1989), Sommer-Larsen, Vedel & Hellsten (1996, paper I in the following).

In paper I we demonstrated that one has to be cautious when comparing results of such simulations with reality when gravity has been softened. This was illustrated by considering, as an example, non-rotating, self-gravitating, isothermal spheres in hydrostatic equillibrium showing that by introducing gravitational softening the structure of such isothermal spheres can be dramatically changed relative to the Newtonian case. This occurs, in qualitative terms, when the radial extent of the spheres is less than or of the order the gravitational softening length, ϵ (in paper I as well as in this paper the one-parameter spline softening proposed by Hernquist & Katz (1989) is used).

It also follows from paper I that for isothermal spheres of a given mass and temperature the size increases with increasing ϵ. In particular, for isothermal spheres of characteristic scale $r \lesssim \epsilon/2$ (and given mass and temperature) it follows from paper I that $r \propto \epsilon^{3/2}$.

In this paper we study the global structure of self-gravitating disks in centrifugal equillibrium for softened gravity. We show that if the characteristic size r of the disks is less than or of the order ϵ, then the size of the disks is significantly larger than for the Newtonian case for a given mass and angular momentum. We also show that if the characteristic size of the disk $r \lesssim \epsilon/2$ then $r \propto \epsilon^{3/4}$ (for a given mass and specific angular momentum distribution with mass).

In section 2 we describe the effects of gravitational softening on the global structure of self-gravitating disks and discuss the results obtained. Finally section 3 constitutes the conclusion.

2 Effects of gravitational softening on the global structure of self-gravitating disks in centrifugal equillibrium

The surface density of a stellar, galactic disk is in general well approximated by a truncated exponential (e.g. Freeman 1970, Van der Kruit 1987)

$$\Sigma(R) = \begin{cases} \Sigma_0 \exp(-R/R_d) \ , & R < R_t \\ 0 \ , & R \geq R_t \end{cases} \tag{1}$$

where Σ_0 is the central surface density, R_d the exponential scale length, R_t the truncation radius ($R_t \sim 4R_d$) and R the radial coordinate in the plane of the disk. Consider first, for a given mass and angular momentum, the one-parameter family of self-gravitating, infinitely flat disks in centrifugal equillibrium described by eq. [1] (with $R_t = 4R_d$) for various values of the gravitational softening length ϵ. As ϵ increases and the gravitational field of the disk becomes increasingly softened one would expect R_d to increase (for a given mass and angular momentum) and hence the characteristic cross section of the disk

$$\sigma_c = \pi R_{rms}^2, \quad \text{where} \quad R_{rms} \equiv < R^2 >^{1/2} \quad , \tag{2}$$

compared to the Newtonian gravity value $\sigma_{c,0}$ - in eq. [2] $< R^2 >$ denotes the mass weighted average of R^2. In Figure 1 $\sigma_c/\sigma_{c,0}$ is shown as a function of the characteristic size of the disk R_{rms} in units of ϵ. As can be seen from the figure, when the characteristic size of the disk is equal to the gravitational softening length, σ_c is about five times larger than $\sigma_{c,0}$ due to the effect of gravitational softening. For $R_{rms} \simeq 0.4\epsilon$ this ratio has increased further to about two orders of magnitude. In the limit $R_{rms} \ll \epsilon$, $\sigma_c/\sigma_{c,0} \propto (R_{rms}/\epsilon)^{-6}$, as can be easily verified.

Clearly, gravitational/hydrodynamical simulations where self-gravitating disks of size comparable to or smaller than the gravitational softening length occur have to be interpreted with care.

In the simple calculations above the mass and angular momentum was assumed fixed, but clearly the specific angular momentum distribution, $j(M)$, where M is the cumulative mass within radius R, changes with ϵ. Indeed, for Newtonian gravity and a given self-gravitating disk in centrifugal equilibrium with surface density $\Sigma(R;\epsilon{=}0)$ and corresponding specific angular

momentum distribution $j(M)$, a disk solution $\Sigma(R;\epsilon)$ with the same mass and $j(M)$ does not necessarily exist for softened gravity with a given non-zero softening length ϵ.

For a disk with surface density given by eq. [1] for Newtonian gravity ($\epsilon = 0$, $\Sigma_0 = R_d = 1$, $R_t = 4R_d$) we were able to obtain disk solutions $\Sigma(R;\epsilon)$ with the same mass and $j(M)$ for $\epsilon \gtrsim R_d$ using a numerical, iterative algorithm. The solutions are shown in Figure 2 as $\epsilon^{3/2}\Sigma$ as a function of $\epsilon^{-3/4}R$ (see below) for $R_d/\epsilon = 2^{-n}, n = 0, 2, .., 8$. The corresponding values of $\sigma_c/\sigma_{c,0}$ are plotted in Figure 1 as filled circles for $n = 0, 1, .., 7$.

For $0 < \epsilon \lesssim R_d$ no solutions could be obtained - we comment further on this below.

2.1 The linear approximation and self-similarity

By an argument similar to the one given in paper I it can be shown that for a system of linear size less than about half the gravitational softening length the gravitational acceleration at $R \lesssim \epsilon/2$ is approximately linear in R:

$$g_\epsilon(R) = -qR + O(R^3), \quad \text{for} \quad R \lesssim \epsilon/2 \quad , \tag{3}$$

where

$$q = \frac{4GM_d}{3\epsilon^3} \quad , \tag{4}$$

G being the gravitational constant and M_d the total mass of the disk. In this limit it is possible to obtain a disk solution $\Sigma(R;\epsilon)$ given a disk $\Sigma(R;0)$ with a corresponding $j(M)$:

By definition

$$j(M) = v_c(R)R \quad , \tag{5}$$

where the circular speed at R is given by (Binney & Tremaine 1987)

$$v_c(R) = R^{1/2}\left[2\pi G\int_0^\infty [\int_0^\infty J_0(kR')\Sigma(R';0)R'dR']\ J_1(kR)kdk\right]^{1/2} \quad , \tag{6}$$

where J_0 and J_1 $(= -J_0')$ are Bessel functions of the first kind, and

$$M(R) = 2\pi\int_0^R \Sigma(R';0)R'dR' \quad . \tag{7}$$

For later use

$$\frac{dj}{dM} = \frac{dj}{dR}\frac{dR}{dM} = \frac{1}{2\pi R\Sigma(R;0)}[v_c(R) + R\frac{dv_c(R)}{dR}] \ . \tag{8}$$

Now, in the linear approximation (assuming $\epsilon \neq 0$)

$$g_\epsilon(R) = -qR, \ \Rightarrow \ v_c(R) = q^{1/2}R \ , \tag{9}$$

so

$$j(R) = q^{1/2}R^2 \ , \tag{10}$$

and

$$\frac{dj}{dM} = \frac{dj}{dR}\frac{dR}{dM} = \frac{2q^{1/2}R}{2\pi R\Sigma(R;\epsilon)} = \frac{q^{1/2}}{\pi\Sigma(R;\epsilon)} \ . \tag{11}$$

Hence

$$\Sigma(R;\epsilon) = (\frac{4GM_d}{3\pi^2})^{1/2} \ \epsilon^{-3/2}(\frac{dj}{dM})^{-1} \ , \tag{12}$$

where, by eq. [10],

$$R(j) = q^{-1/4}j^{1/2} = (\frac{3}{4GM_d})^{1/4}\epsilon^{3/4}j^{1/2} \ . \tag{13}$$

Given a $\Sigma(R;0)$ eqs. [6]-[8], [12] and [13] can be used to determine the corresponding $\Sigma(R;\epsilon)$. It is easy to show that the solutions are self-similar:

$$\Sigma(R;\alpha\epsilon) = \alpha^{-3/2} \ \Sigma(\alpha^{-3/4}R;\epsilon) \ . \tag{14}$$

For $\Sigma(R;0)$ given by eq. [1], $\epsilon^{3/2}\Sigma(R;\epsilon)$ as a function of $\epsilon^{-3/4}R$ is shown in Figure 2 as a solid line. The solutions obtained previously for $R_d/\epsilon = 2^{-n}, n = 0, 2, .., 8$, as displayed in Figure 2, converge towards this limiting solution as $R_d/\epsilon \rightarrow 0$, as expected.

Given that in the linear approximation a solution $\Sigma(R;\epsilon)$ with the same mass and specific angular momentum distribution as the Newtonian disk $\Sigma(R;0)$ can be obtained (eq. [12]) it is not surprising that such solutions in general can be obtained for R_d/ϵ less than some critical value. But, as described in the previous subsection, one should not expect to be able to obtain such solutions for all values of R_d/ϵ.

In the simplified Tree-SPH galaxy formation simulations described in paper I self-gravitating, quasi-isothermal gas systems of characteristic size

$r \lesssim \epsilon/2$ occur at t = 3.2 Gyr in three, otherwise identical, simulations with ϵ = 1.5, 3.0 and 6.0 kpc respectively. As discussed in paper I the systems become increasingly pressure supported relative to rotational support as ϵ increases.

For systems of equal mass and temperature it follows from eqs. [29] and [31] of paper I that one would expect $r \propto \epsilon^{3/2}$ for pressure supported systems. Likewise it follows from eq. [14] above that for rotationally supported systems of equal mass and $j(M)$ one would expect $r \propto \epsilon^{3/4}$. In Figure 3 we have plotted the half-mass radii of the most massive, self-gravitating system at t = 3.2 Gyr in the three simulations as a function of ϵ. As can be seen from this Figure $r_{1/2}(\epsilon)$ increases approximately as $\epsilon^{3/4}$ as ϵ increases from 1.5 to 3.0 kpc and approximately as $\epsilon^{3/2}$ as ϵ increases further from 3.0 to 6.0 kpc in good agreement with the increasing degree of pressure support found in paper I.

3 Conclusion

In hydrodynamical/gravitational simulations involving gravitational softening an adverse effect of the softening is to make simulated, self-gravitating disks artificially large by a significant factor if the characteristic size, r, of the simulated disks is less than or comparable to the gravitational softening length, ϵ. In such cases the outcome of the simulations should be interpreted with due care.

If $r \lesssim \epsilon/2$ then the scaling relation $r \propto \epsilon^{3/4}$ holds for disks with a given mass and specific angular momentum distribution $j(M)$.

Finally, the theoretical results obtained in this paper and paper I are compared with the results of numerical, Tree-SPH simulations and qualitative agreement is found.

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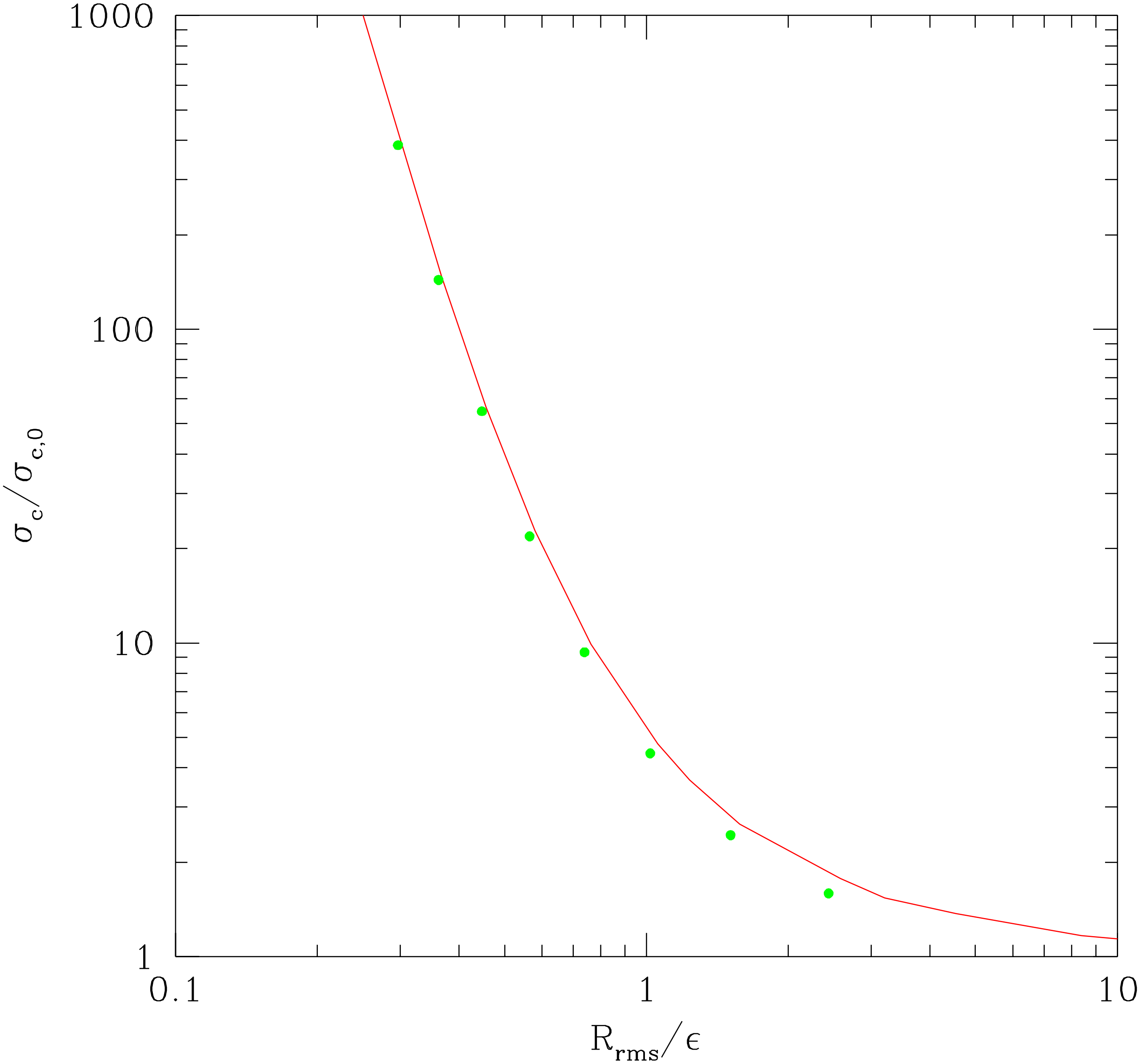
Figure captions

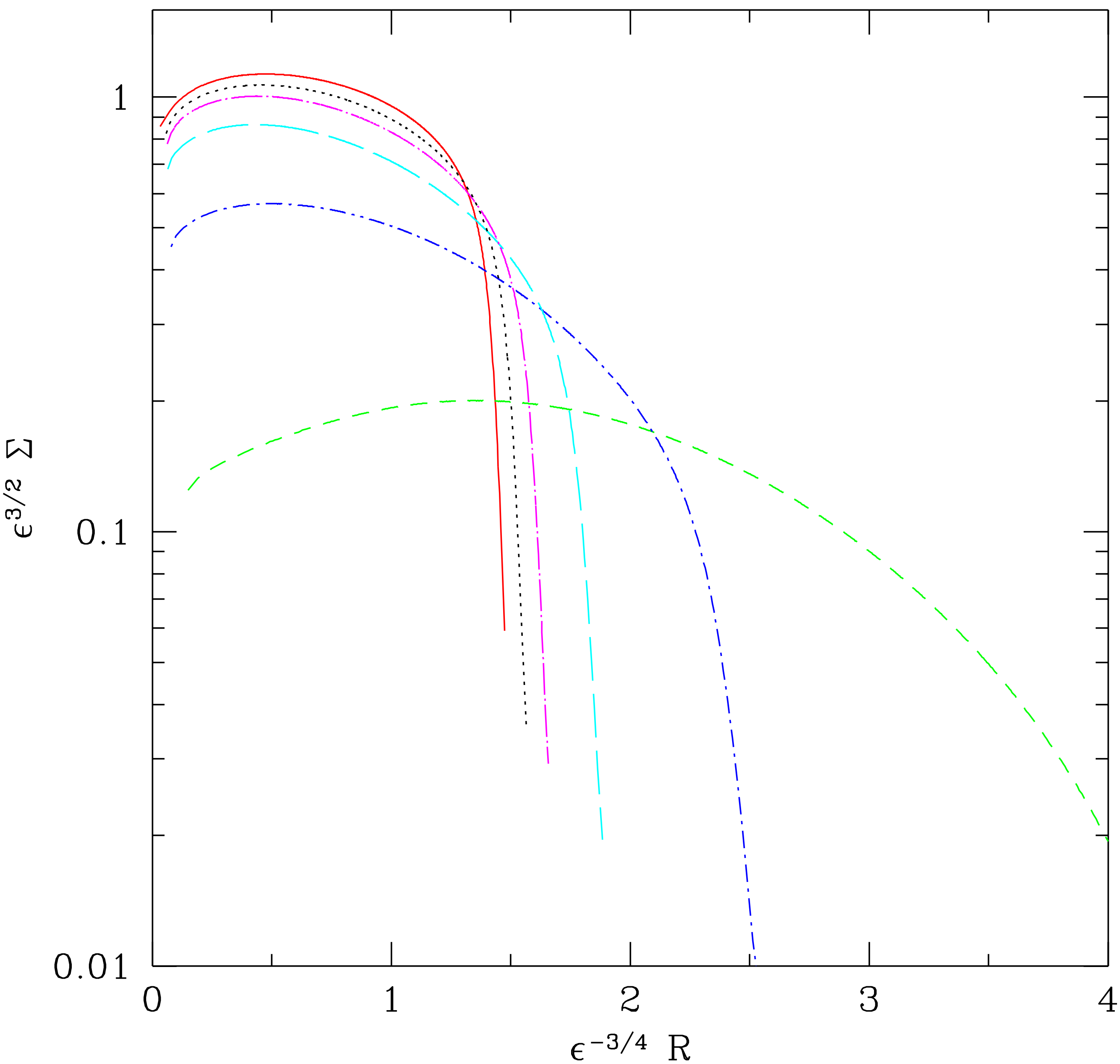
Figure 1: The ratio of the characteristic cross section of an exponential disk for softened gravity to that of a Newtonian exponential disk for a given mass and angular momentum (solid line). The dots show the same quantities for solutions of given mass and specific angular momentum distribution $j(M)$.

Figure 2: The surface density distribution in the linear approximation of a disk of the same mass and specific angular momentum distribution, $j(M)$, as a Newtonian, exponential disk with $R_t = 4R_d$, displayed as $\epsilon^{3/2}\Sigma$ vs. $\epsilon^{-3/4}R$ (solid line). Other curves: Solutions of the same mass and $j(M)$ for $R_d/\epsilon = 2^{-n}, n = 0, 2, .., 8$ also displayed as $\epsilon^{3/2}\Sigma$ vs. $\epsilon^{-3/4}R$. The solutions converge monotonically toward the limiting solution shown by the solid line as n increases.

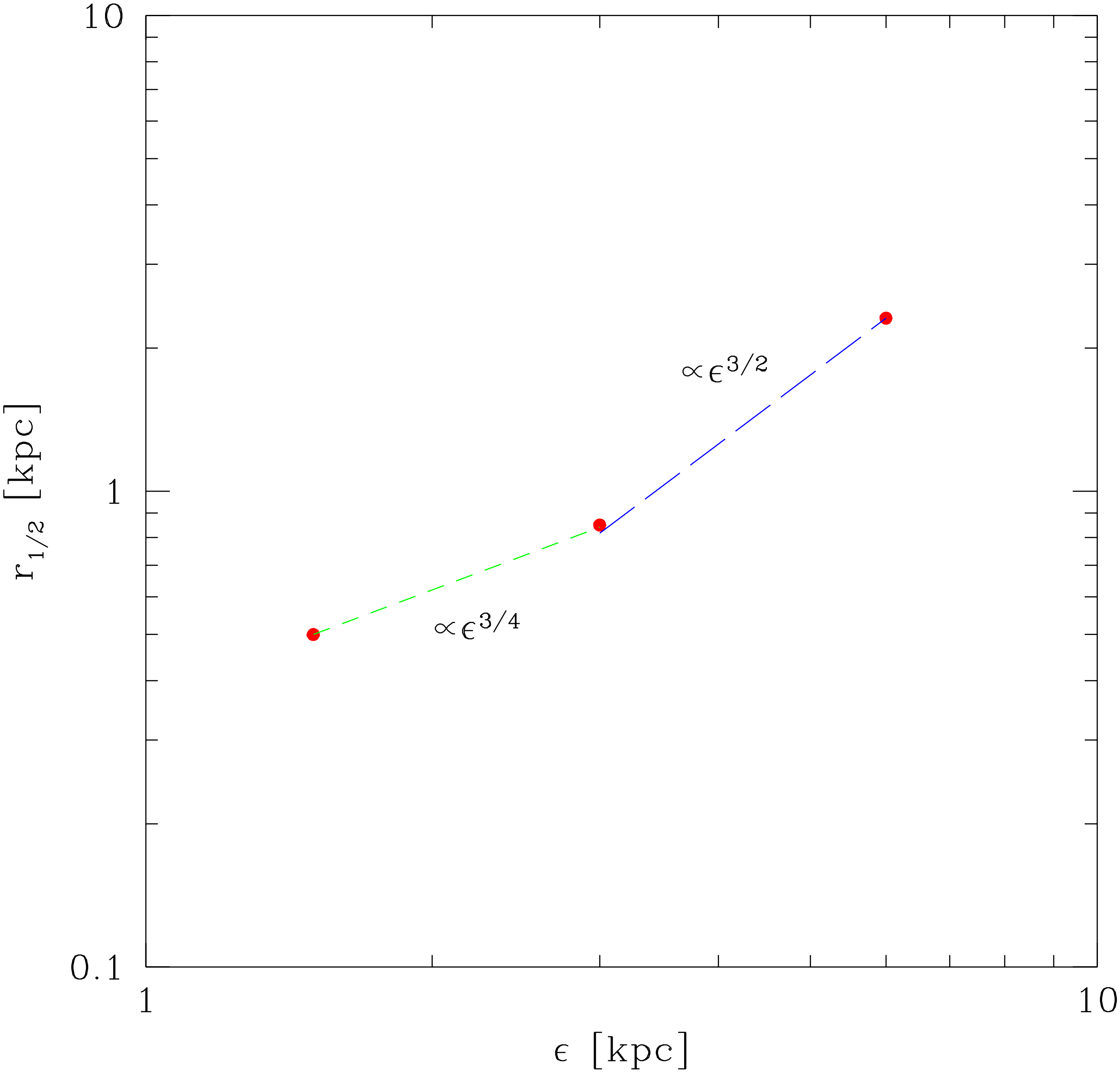
Figure 3: Half-mass radius as a function of ϵ for the most massive, dense gas clumps at $t = 3.2$ Gyr occurring in the simulations described in paper I. Also shown are power law relations of logarithmic slope 3/4 and 3/2 respectively.



1000
100
10
1
0.1
1
10
$\sigma_c/\sigma_{c,0}$
R_{rms}/ϵ



$\epsilon^{3/2}\ \Sigma$
1
0.1
0.01
0
1
2
3
4
$\epsilon^{-3/4}\ R$


10
1
0.1
1
10
$r_{1/2}$ [kpc]
ϵ [kpc]
$\propto \epsilon^{3/2}$
$\propto \epsilon^{3/4}$